MET 4501 Spring 2025: Practice Exam 1 Name: SOLUTION

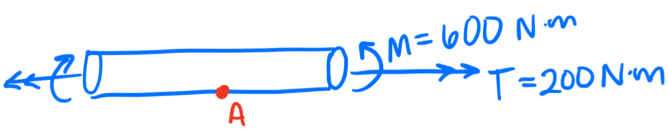
Section 1: Multiple Choice (4 points each, 40 points total)

1. Which of the following best describes the purpose of using factors of safety in mechanical design?
   - O To optimize material usage
   - ● To ensure that a design can withstand more load than expected
   - O To reduce production costs
   - O To minimize the weight of the design
2. Which of the following best describes the Maximum Normal Stress (MNS) theory?
   - O Failure occurs when the maximum shear stress exceeds a critical value.
   - ● Failure occurs when the maximum normal stress exceeds the ultimate tensile strength.
   - O Failure occurs when the strain energy density exceeds the failure threshold.
   - O Failure occurs when the von Mises stress exceeds the yield strength.
3. Which failure criterion is typically used for ductile materials in static loading?
   - O Modified Mohr theory
   - ● Distortion Energy Theory
   - O Maximum Principal Stress Theory
   - O Brittle Coulomb-Mohr theory
4. When analyzing a beam subjected to torsion, which of the following stresses must be evaluated?
   - O Normal stress
   - ● Shear stress
   - O Bending stress
   - O Hoop stress
5. Which of the following best describes the purpose of conceptual design?
   - O To generate detailed design drawings
   - ● To develop initial ideas and possible solutions
   - O To select final materials
   - O To test prototypes
6. The critical stress intensity factor, K_{IC}, is also known as the:
   - O Tensile strength
   - O Fracture energy
   - ● Fracture toughness
   - O Geometry
7. Which of the following is NOT an assumption of static failure analysis?
   - O Material exhibits linear elastic behavior up to failure.
   - ● Stresses are time dependent.
   - O The loading is applied gradually and remains constant.
   - O Failure is governed by a single, instantaneous load event.

8. What does the slope of the linear portion of a stress-strain curve represent for a material?
   - O Yield strength
   - ● Modulus of elasticity (Young's modulus)
   - O Ultimate tensile strength
   - O Toughness
9. Which of the following failure criteria is most appropriate for analyzing the failure of a ductile material under static loading?
   - O Modified Mohr (MM) theory
   - ● Maximum Shear Stress (MSS) Theory
   - O Maximum Normal Stress (MNS) Theory
   - O Brittle Coulomb-Mohr (BCM) theory
10. In a beam subject to pure bending, which type of stress is experienced along the length of the beam?
    - O Shear stress
    - ● Normal stress
    - O Torsional stress
    - O Hoop stress

Section 2: Problem-Solving (60 points total)

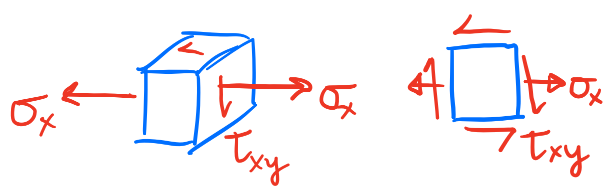
11. (20 points) A circular shaft with a diameter of 30 mm is subjected to a bending moment of 600 N·m and a torsional moment of 200 N·m. The shaft is made of a brittle material with an ultimate strength in tension of 100 MPa, and an ultimate strength in compression of 400 MPa. Using the Modified-Mohr theory, determine whether the shaft will fail. Show all calculations.



STRESS DUE TO M IS MAX (TENSION) ON BOTTOM
STRESS DUE TO T IS MAX ON EXTERIOR
↳ CHOOSE POINT A TO ANALYZE

$$\sigma_x = \frac{My}{I} = \frac{(600\ N\cdot m)(0.015\ m)}{3.98\times10^{-8}\ m^4} = 226\times10^6\ Pa = 226\ MPa$$

$$I = \frac{\pi r^4}{4} = \frac{\pi(15\ mm)^4}{4} = \frac{\pi(0.015\ m)^4}{4} = 3.98\times10^{-8}\ m^4$$



$$\tau_{xy} = \frac{Tr}{J} = \frac{(200\ N\cdot m)(0.015\ m)}{7.95\times10^{-8}\ m^4} = 37.7\times10^6\ Pa = 37.7\ MPa$$

$$J = \frac{\pi r^4}{2} = \frac{\pi(0.015\ m)^4}{2} = 7.95\times10^{-8}\ m^4$$

$$\sigma_{A,B} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$= 113\ MPa \pm \sqrt{12769+1421}\ MPa$$

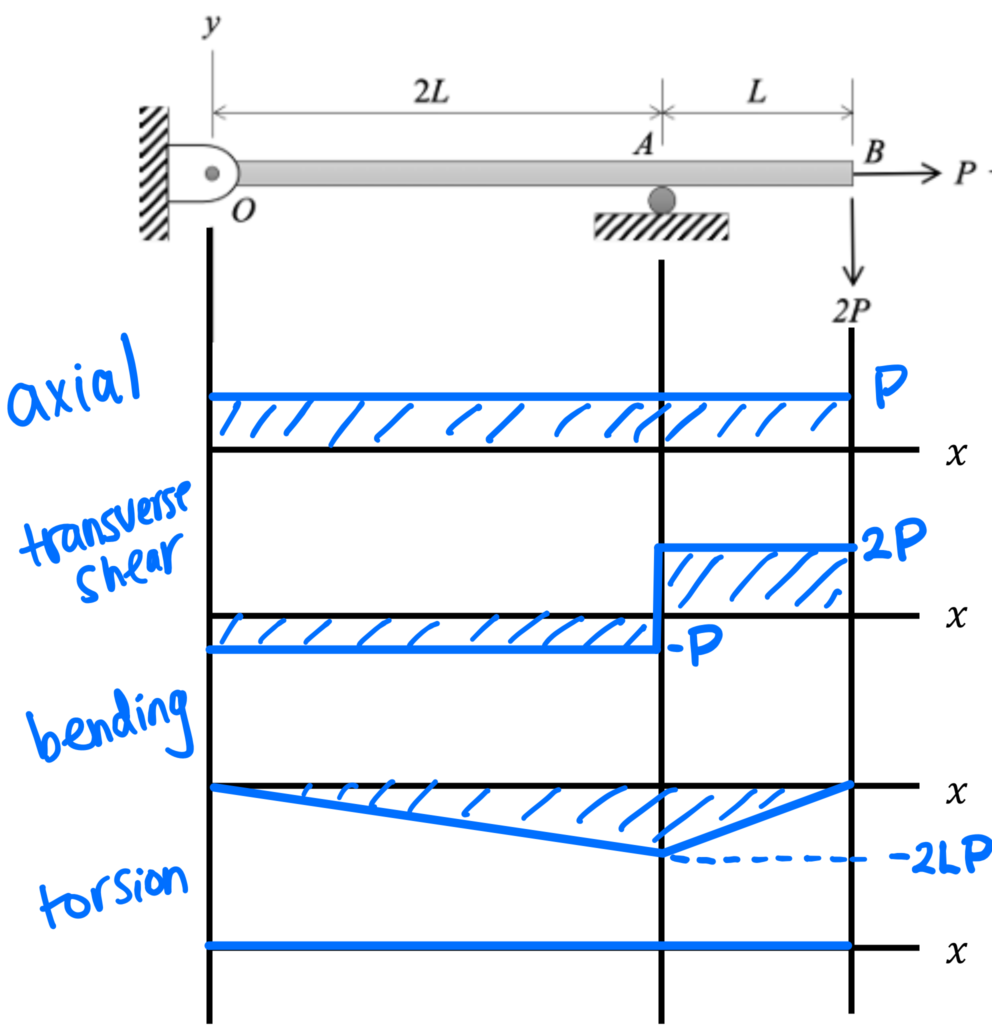
$$= 113 \pm 119$$

$$\sigma_A = 232\ MPa,\ \sigma_B = -6\ MPa$$

MM: $\sigma_A \geq 0 \geq \sigma_B$ & $|\sigma_A| \geq |\sigma_B|$

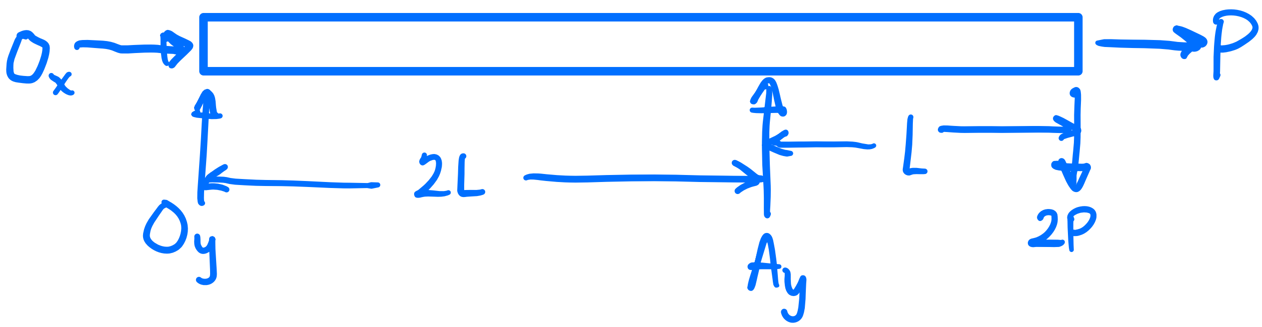
$$n = \frac{S_{ut}}{\sigma_A} = \frac{100\ MPa}{232\ MPa} = \underline{0.43} \leftarrow \text{FAILURE PREDICTED!}$$

12. (40 points total) Rod *OAB* has length $3L$ and diameter $d = L/6$.
The rod is supported by a pin joint at *O* and by a roller at *A*.
Axial load *P* and transverse load *2P* act at *B*.
The rod is made of a ductile material with yield strength S_y.
Determine the following:
    a) (5 points) Solve for the reactions at *O* and *A*.
    b) (10 points) Sketch and label diagrams of the internal loads on the axes provided.
    c) (5 points) Identify the critical cross-section of rod *OAB*.
    d) (5 points) Identify the critical element on the cross-section identified in part (c). You may use the attached Combined Stress Analysis Worksheet to aid your analysis.
    e) (5 points) Show the state of stress on a stress element for the critical element.
    f) (10 points) The factor of safety for the critical element in terms of variables *P*, *L*, and S_y. Use both the distortion energy (DE) and maximum shear stress (MSS) failure theories. If needed, axes to draw Mohr's circle are provided on the next page.



[Problem 12 continued]

(a)



$\Sigma F_x = 0 \Rightarrow \boxed{O_x = -P}$

$\Sigma M_O = 0 \Rightarrow A_y(2L) - 2P(3L) = 0$

$A_y = \dfrac{6PL}{2L} \Rightarrow \boxed{A_y = 3P}$

$\Sigma F_y = 0 \Rightarrow O_y + A_y - 2P = 0 \Rightarrow \boxed{O_y = -P}$

(b) SEE ABOVE PAGE.

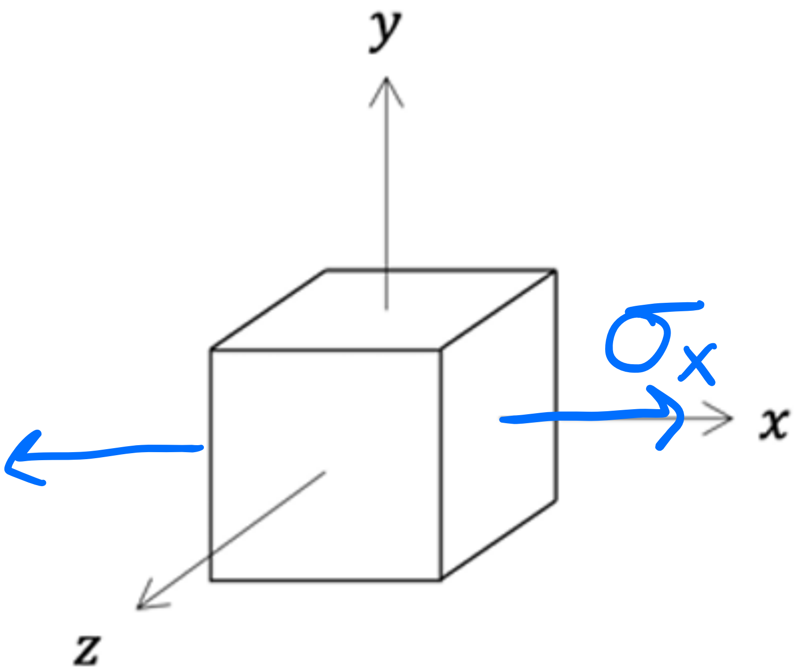
(c) THE CRITICAL CROSS-SECTION IS JUST TO THE RIGHT OF A. (MAX TRANS. SHEAR & BENDING)

(d) SEE ATTACHED WORKSHEET. CRITICAL ELEMENT IS LOCATED ON TOP OF BEAM.

(e) SEE NEXT PAGE

[Problem 12 continued]

Stress element for the critical element:



$$\sigma_x = \frac{P}{A} + \frac{M_z c}{I}$$

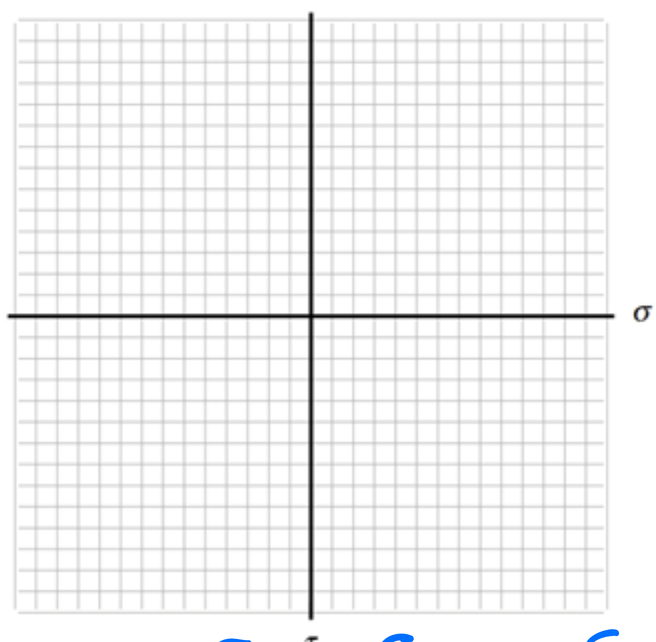
$$= \frac{P}{\frac{\pi}{4}\left(\frac{L}{6}\right)^2} + \frac{2PL\left(L/12\right)}{\frac{\pi}{64}\left(L/6\right)^4}$$

$$= \frac{144P}{\pi L^2} + \frac{64(6)^4 PL^2}{6\pi L^4}$$

$$= \frac{144P}{\pi L^2} + \frac{13824P}{\pi L^2}$$

$$= 4446\frac{P}{L^2}$$

Axes to draw Mohr's circle:



(f) $\sigma_1 = \sigma_x = 4446\frac{P}{L^2}$

$\sigma_2 = 0$

$\sigma_3 = 0$

DE: $n_y = \frac{S_y}{\sigma'}$

$$\sigma' = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{2}}$$

$$= \sqrt{\frac{\sigma_x^2+\sigma_x^2}{2}} = \sigma_x$$

$$\boxed{n_y = \frac{S_y}{\sigma_x} = \frac{S_yL^2}{4446P}}$$

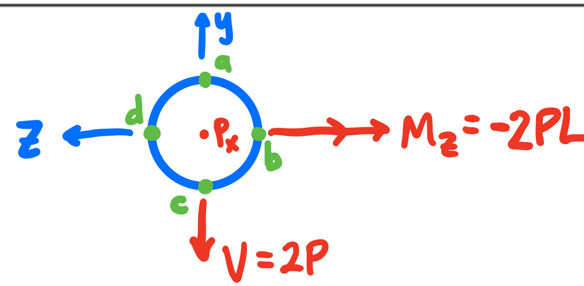
MSS: $n_y = \frac{S_y}{\tau_{max}} = \frac{S_y}{\sigma_1-\sigma_3} = \frac{S_y}{\sigma_x}$

$$\boxed{n_y = \frac{S_yL^2}{4446P}}$$

same

In this box,

- Draw the critical cross-section
- Identify and label the potential locations for the critical element(s) (e.g. top, bottom, left, right, and center)



$P_x = P$ AXIAL LOAD OUT OF PAGE

Potential location of critical element		a	b	c	d	
Internal load	Axial	$\sigma_x = \frac{P}{A}$	$\sigma_x = \frac{P}{A}$	$\sigma_x = \frac{P}{A}$	$\sigma_x = \frac{P}{A}$	
	Torsion	none →				
	Transverse shear	0	$\tau_{xy} = \frac{4V}{3A} = \frac{-8P}{3A}$	0	$\tau_{xy} = \frac{4V}{3A} = \frac{-8P}{3A}$	
	Bending	$\sigma_x = \frac{M_z c}{I}$ (tension)	0	$\sigma_x = \frac{-M_z c}{I}$ (compression)	0	
Stress element		y, x, z; $\boxed{\sigma_x = \frac{P}{A} + \frac{M_z c}{I}}$	y, x, z; $\tau_{xy} = \frac{8P}{3A}$	y, x, z; $\sigma_x = \frac{P}{A} - \frac{M_z c}{I}$	y, x, z; $\tau_{xy} = \frac{8P}{3A}$	y, x, z